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EVOLUTION OF THE SHAPE OF A POLYMER SUBJECTED TO A FORCE.(U)

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UNCLASSIFIED MRC-TSR-2150 NL

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MRC Technical Summary Report #2150

EVOLUTION OF THE SHAPE OF A POLYMER SUBJECTED TO A FORCE

M. Renardy

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December 1980

(Received November 5, 1980)

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North Carolina 27709

UNIVERSITY OF WISCONSIN-MADISON
MATHEMATICS RESEARCH CENTER

EVOLUTION OF THE SHAPE OF A POLYMER SUBJECTED TO A FORCE

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ABSTRACT

We consider the nonlinear Volterra integrodifferential equation

$$-\mu\dot{y} = \int_{-\infty}^{t} a(t-s)\left(\frac{y^3(t)}{y^2(s)} - y(s)\right)ds - f(t)y^{\alpha}$$

where $a(u) = \sum_{i=1}^{N} K_i e^{-\lambda_i u}$, $0 < \alpha < 3$, and $f: \mathbb{R} \to \mathbb{R}$ is a given function. We study the existence of solutions and their asymptotic behavior as $t \to \infty$ for certain classes of functions f. The main result is that for each f satisfying $\lim_{t \to -\infty} e^{-\sigma t} f(t) = 0$ for some $\sigma > 0$ and $f(t) = 0$ for $t \geq t_0$ there exists a unique solution $y(t)$ satisfying $\lim_{t \to -\infty} y(t) = 1$. Moreover, for this solution $\lim_{t \to +\infty} y(t)$ exists. This holds both for $\mu > 0$ and $\mu = 0$. It is further shown that the condition $f(t) = 0$ for $t \geq t_0$ can not be replaced by exponential decrease unless f is small or $\alpha = 1$.

AMS (MOS) Subject Classifications: 34A15, 34D05, 34D20, 34D40, 45J05, 76A10

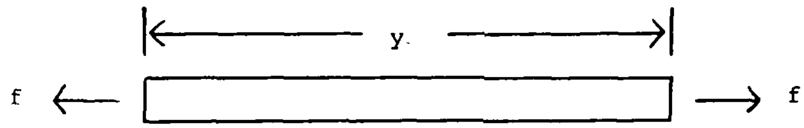
Key Words: Materials with memory, nonlinear Volterra integrodifferential equations, asymptotic behavior, Liapounov functions, implicit function theorem

Work Unit Number 1 (Applied Analysis)

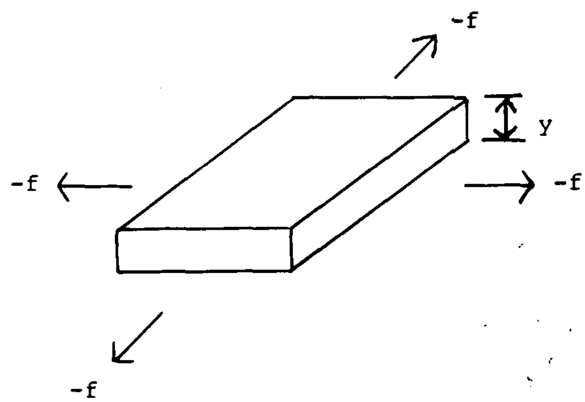
Sponsored by the United States Army under Contract No. DAAG29-80-C-0041 and Deutsche Forschungsgemeinschaft.

SIGNIFICANCE AND EXPLANATION

The equation describes the evolution of the shape of a filament or a sheet of a polymeric liquid subjected to an external force $f(t)$. y denotes the length of the filament or the thickness of the sheet respectively, and μ is a viscosity constant which can either be positive or zero. The exponent α depends on the physical situation that is being considered.



$\alpha = 2$



$\alpha = \frac{1}{2}$

We consider the length at $t = -\infty$ as known and investigate the evolution. It is shown that for a physically realistic class of functions f there exists a unique solution, and that the length approaches a new stationary value at $t = \infty$ (which is in general different from the value at $t = -\infty$).

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

EVOLUTION OF THE SHAPE OF A POLYMER SUBJECTED TO A FORCE

M. Renardy

1. INTRODUCTION

In [1] Lodge, Mc Leod and Nohel have studied the history value problem for the nonlinear Volterra integrodifferential equation

$$-\mu\dot{y}(t) = \int_{-\infty}^{t} a(t-s)\, F(y(t),y(s))ds$$

This equation arises as a mathematical model for the stretching of a filament or a sheet of a molten polymer which is assumed to be spatially homogeneous. In this case y denotes the length of the polymer, μ is a parameter related to the viscosity,

$$F(y,z) = \frac{y^3}{z^2} - z$$

and

$$a(u) = \sum_{i=1}^{N} K_i e^{-\lambda_i u}$$

with certain positive constants K_i and λ_i. It was assumed in [1] that, for $t \le 0$, $y(t) = g(t)$ was given, $g(-\infty) = 1$, and g nondecreasing. A general class of functions a and F, which includes those mentioned, was considered. One of the results of [1] is that under these conditions the history value problem has a unique solution, which is nonincreasing for $t \ge 0$, and approaches a constant strictly greater than 1 as $t \to \infty$. This paper deals with a closely related problem. We consider the elongation of a polymer satisfying the same constitutive law, but rather than prescribing the length history we prescribe the force acting on the polymer, which causes the elongation, and consider only the length at $t = -\infty$ as known. Unlike [1], we explicitly use the form of F and a as given above.

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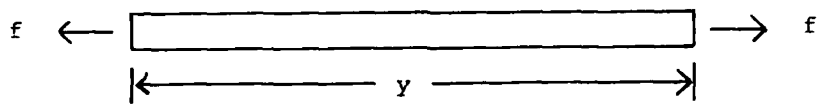
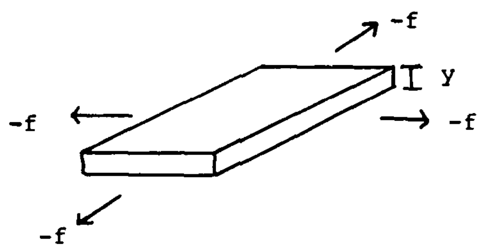
It can be derived (for an explanation of the physical principles see [2]) that the elongation of the polymer is now described by the equation

$$-\mu\dot{y} = \int_{-\infty}^{t} a(t-s)\left(\frac{y^3(t)}{y^2(s)} - y(s)\right)ds - f(t)y^\alpha \tag{1.1}$$

The exponent α depends on the physical situation. One case of interest is that of a filament pulled at its ends



In this case $\alpha = 2$. Another physically interesting situation is a sheet of polymer pulled in two directions.



In this case y denotes the thickness of the sheet and $\alpha = \frac{1}{2}$. Our analysis holds in general for $0 < \alpha < 3$. The main result we are going to prove is that for each f satisfying $\lim_{t \to -\infty} e^{-\sigma t} f(t) = 0$ for some $\sigma > 0$ there exists a unique solution of the equation satisfying $\lim_{t \to -\infty} y(t) = 1$. (Theorem 3.1.). Moreover, if $f = 0$ for t greater than some $t_0 < \infty$, then this solution approaches a constant as $t \to +\infty$. Counterexamples show that this last condition on f cannot be replaced by exponential decrease as $t \to +\infty$ except for small f (Theorem 2.4.).

2. SOLUTIONS FOR SMALL FORCES

Let us consider equation (1.1), where $0 < \alpha < 3$, $a(u) = \sum_{i=1}^{N} K_i e^{-\lambda_i u}$, and $\mu > 0$. This equation can be reduced to a system of ODE's in the following way. We put

$$g_i(t) = \int_{-\infty}^{t} K_i e^{-\lambda_i (t-s)} \frac{1}{y^2(s)} ds$$

$$h_i(t) = \int_{-\infty}^{t} K_i e^{-\lambda_i (t-s)} y(s) ds$$

Then (1.1.) reads

$$\begin{aligned} -\mu\dot{y} &= \sum_{i=1}^{N} (g_i y^3 - h_i) - f(t)y^\alpha \\ \dot{g}_i &= -\lambda_i g_i + \frac{K_i}{y^2} \\ \dot{h}_i &= -\lambda_i h_i + K_i y \end{aligned} \qquad (2.1.)$$

If we put $\gamma_i = g_i y^2$, $\delta_i = \frac{h_i}{y}$, we obtain

$$\begin{aligned} -\mu\dot{y} &= \sum_{i=1}^{N} (\gamma_i - \delta_i)\, y - f(t)y^\alpha \\ \dot{\gamma}_i &= -\lambda_i\gamma_i + K_i - \frac{2}{\mu}\gamma_i \sum_j (\gamma_j - \delta_j) + \frac{2}{\mu}\gamma_i f(t) y^{\alpha-1} \\ \dot{\delta}_i &= -\lambda_i\delta_i + K_i + \frac{1}{\mu}\delta_i \sum_j (\gamma_j - \delta_j) - \frac{1}{\mu}\delta_i f(t) y^{\alpha-1} \end{aligned} \qquad (2.2.)$$

Both forms (2.1.) and (2.2.) will be used in the following. Clearly, if $f = 0$, we have the stationary solution $y = 1$, $g_i = h_i = \frac{K_i}{\lambda_i} = \gamma_i = \delta_i$.

<u>LEMMA 2.1</u> The linearization of (2.1) (or (2.2) at the stationary solution $y = 1$, $g_i = h_i = \frac{K_i}{\lambda_i}$ has zero as a simple eigenvalue. All the other eigenvalues have negative real parts.

<u>PROOF:</u> Clearly, (2.1) and (2.2) give the same eigenvalues. Let us consider (2.1). We obtain the following matrix of the linearization

$$\begin{pmatrix}
-\sum\limits_{i=1}^{N} \frac{3K_i}{\mu\lambda_i} & -\frac{1}{\mu} & -\frac{1}{\mu} & \cdots & -\frac{1}{\mu} & \frac{1}{\mu} & \frac{1}{\mu} & \cdots & \frac{1}{\mu} \\
-2K_1 & -\lambda_1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
-2K_2 & 0 & -\lambda_2 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
-2K_N & 0 & 0 & \cdots & -\lambda_n & 0 & 0 & \cdots & 0 \\
K_1 & 0 & 0 & \cdots & 0 & -\lambda_1 & & \cdots & 0 \\
K_2 & 0 & 0 & \cdots & 0 & 0 & -\lambda_2 & \cdots & 0 \\
\vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
K_N & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & -\lambda_n
\end{pmatrix}$$

This yields the characteristic polynominal.

$$P(\lambda) = \prod_j (-\lambda_j - \lambda)^2 \left(-\sum_i \frac{3K_i}{\mu\lambda_i} - \lambda - \sum_i \frac{3K_i}{\mu(-\lambda_i - \lambda)}\right)$$

Thus N eigenvalues are given by $\lambda = -\lambda_i$, the remaining $N + 1$ eigenvalues are the zeros of the last factor. Obviously one of these is zero, and it is simple. It remains to be proved that all the remaining solutions have negative real parts. Assume the contrary, i.e. $\mathrm{Re}\lambda \geq 0$, $\lambda \neq 0$ and

$$-\sum_i \frac{3K_i}{\mu\lambda_i} - \lambda - \sum_i \frac{3K_i}{\mu(-\lambda_i - \lambda)} = 0$$

When we take the real part, this implies

$$-\sum_i \frac{3K_i}{\mu\lambda_i} - \mathrm{Re}\,\lambda + \sum_i \frac{3K_i\,(\lambda_i+\mathrm{Re}\lambda)}{\mu[(\lambda_i+\mathrm{Re}\lambda)^2 + (\mathrm{Im}\lambda)^2]} = 0$$

It is, however, easy to see that the left hand side is strictly negative, whence we have a contradiction.

Let C_T^n denote the space of all T-periodic C^n-functions $\mathbb{R} \to \mathbb{R}$. The following lemma holds:

<u>LEMMA 2.2</u> If $f_0(t) \in C_T^n$ has sufficiently small norm, then there exists a one-parameter of constants C and functions $Y = (y,\gamma_i,\delta_i) \in (C_T^{n+1})^{2N+1}$ such that Y satisfies equation (2.2) with $f = f_0 - C$.

<u>PROOF</u>: Obviously (2.2) with $f = f_0 - C$ can be written in the form $G(Y,c,f_0) = 0$, where G is a smooth function mapping a neighborhood of $Y = Y_0 = (1, \frac{K_i}{\lambda_i}, \frac{K_i}{\lambda_i})$, $c = 0$, $f_0 = 0$ in $(C_T^{n+1})^{2N+1} \times \mathbb{R} \times C_T^n$ into $(C_T^n)^{2N+1}$.

It is a straightforward consequence of lemma 2.1 that the Frechet derivative $D_Y G(Y_0,0,0)$ has a one-dimensional kernel and its range has codimension 1. Moreover, one easily sees from (2.2) that $D_c G(Y_0,0,0)$ is not in the range of $D_Y G(Y_0,0,0)$, i.e. $D_{(Y,c)} G(Y_0,0,0)$ is onto and has a one-dimensional kernel. The lemma now follows from the implicit function theorem.

<u>DEFINITION 2.3</u> Let $X_n^\sigma = \{f \in C^n(\mathbb{R},\mathbb{R}) \mid \lim_{|t|\to\infty} e^{\sigma|t|} f^{(k)}(t) = 0 \text{ for } k = 0,1,\ldots,n\}$

A natural norm in X_n^σ is

$$\|f\| = \sum_{k=0}^{n} \sup_{t\in\mathbb{R}} |e^{\sigma|t|} f^{(k)}(t)|$$

Moreover, let $Y_n^\sigma = \{f \in C^n(\mathbb{R},\mathbb{R}) \mid \lim_{|t|\to\infty} e^{\sigma|t|} f^{(k)}(t) = 0 \text{ for } k = 1,\ldots,n,$

$$\lim_{t\to -\infty} e^{-\sigma t} f(t) = 0\}$$

A natural norm in Y_n^σ is

$$\|f\| = \sum_{k=1}^{n} \sup_{t\in\mathbb{R}} |e^{\sigma|t|} f^{(k)}(t)| + \sup_{t\leq 0} |e^{-\sigma t} f(t)| + \sup_{t\geq 0} |e^{\sigma t}(f(t) - f(\infty)|$$

$$+ |f(\infty)|$$

<u>Theorem 2.4</u> Let again Y denote $(y,\gamma_1,\gamma_2,\ldots,\gamma_n,\delta_1,\ldots\delta_n)$ and $Y_0 = (1,\frac{K_i}{\lambda_i},\frac{K_i}{\lambda_i})$. Let $\sigma > 0$ be small enough (smaller than all the real parts of the non-zero eigenvalues of the linearization). Then the following holds: If $f \in X_n^\sigma$ has sufficiently small norm, then (2.2) has a solution Y satisfying $Y - Y_0 \in Y_{n+1}^\sigma \times (X_{n+1}^\sigma)^{2N}$. Y depends smoothly on f.

<u>PROOF</u>: When we put $Y - Y_0 = Z$, equation (2.2) can be written in the form $G(Z,f) = 0$, and G is a smooth mapping of $(Y_{n+1}^\sigma \times (X_{n+1}^\sigma)^{2N}) \times X_n^\sigma$ into $(X_n^\sigma)^{2N+1}$. Moreover, it can easily be concluded from lemma 2.1 that the *linearization* $D_Z G(0,0)$ *is an isomorphism. The implicit function theorem* implies the result.

3. GLOBAL BEHAVIOUR OF SOLUTIONS FOR LARGE f

THEOREM 3.1 Let $f: \mathbb{R} \to \mathbb{R}$ be continuous and such that $\lim_{t\to -\infty} e^{-\sigma t} f(t) = 0$ ($\sigma>0$ small enough), $f(t) = 0$ for $t \geq t_0$. For every such f, equation (2.2) has a unique solution satisfying $\lim_{t\to -\infty} y(t) = 1$, $\lim_{t\to -\infty} \gamma_i = \lim_{t\to -\infty} \delta_i = \frac{K_i}{\lambda_i}$. This solution exists globally for all times t and $\lim_{t\to +\infty} y(t)$ exists and is strictly greater that zero.

PROOF: If t_1 is chosen large enough, $e^{-\sigma t} f(t)$ becomes small on $(-\infty, -t_1)$ and one can use an implicit function argument analogous to theorem 2.4 to prove the existence of a solution on $(-\infty, -t_1)$. This solution is unique in the class of solutions approaching the limiting values at $t = -\infty$ at a rate of $e^{\sigma t}$. However, if a solution tends to these limits at all, it can be seen from the last two equations of (2.2) and the implicit function theorem that γ_i and δ_i tend to their limiting values at a rate of $e^{\sigma t}$. The first equation then implies that y approaches its limiting value at the same rate. Hence the solution is actually unique in the class of all solutions approaching the prescribed limits at $t = -\infty$ as claimed in the theorem.

We now continue this solution to the right, and we have to make sure that it does not blow up in a finite time. For that purpose it is more convenient to consider (2.1) rather that (2.2). From the second and third equation we see that as long as y stays positive, g_i and h_i have a positive lower bound for all finite times, which is independent of y. Hence, if y becomes too large, $g_i y^3$ will dominate over fy^α and also over h_i (since this is less than some constant times $\max_{(-\infty,t]} y(\tau)$). Analogously, if y becomes too small, h_i will dominate over fy^α and $g_i y^3$. Hence y cannot go to zero or infinity in finite time, whence we find global existence.

Let now $t > t_0$. Then $f = 0$, and using (2.2) again, we find

$$\sum_{i=1}^{N} \left(\frac{\mu}{2} \frac{\alpha_i \dot{\alpha}_i}{\alpha_i + \frac{K_i}{\lambda_i}} + \mu \frac{\beta_i \dot{\beta}_i}{\beta_i + \frac{K_i}{\lambda_i}} \right)$$

$$= - \sum_{i=1}^{N} \left(\frac{\lambda_i \mu}{2} \frac{\alpha_i^{\,2}}{\alpha_i + \frac{K_i}{\lambda_i}} + \lambda_i \mu \frac{\beta_i^{\,2}}{\beta_i + \frac{K_i}{\lambda_i}} \right) - \left(\sum_{i=1}^{N} (\alpha_i - \beta_i) \right)^2 \qquad (3.1)$$

Here we have put $\alpha_i = \gamma_i - \frac{K_i}{\lambda_i}$, $\beta_i = \delta_i - \frac{K_i}{\lambda_i}$. As we know that γ_i and δ_i stay positive, the denominators $\alpha_i + \frac{K_i}{\lambda_i}$, $\beta_i + \frac{K_i}{\lambda_i}$ are always positive, and the left side of the equation (3.1) is thus the derivative of a positive definite function that decreases along trajectories. As an immediate consequence we obtain that α_i and β_i tend to 0 exponentially for $t \to \infty$. One easily concludes from (2.2) that y approaches a constant.

<u>COROLLARY 3.2</u> If f is always non-negative and not identically zero, then $y(\infty) > y(-\infty)$, if f is always non-positive and not identically zero, then $y(\infty) < y(-\infty)$.

<u>PROOF</u>: Assume $f \geqslant 0$, the other case is analogous. It is immediate from the integral equation (1.1) that $f \geqslant 0$ implies $y \geqslant 1$ for all t. Moreover, if $f \not\equiv 0$, there must be some t^* such that $y(t^*) > 1$.

Let now $z(t) = \min\limits_{\tau \in [t^*,t]} y(\tau)$. Then (1.1) implies that

$$\left(\frac{d}{dt}\right)_+ z(t) \geqslant \min \left(0, -\frac{1}{\mu} \int_{-\infty}^{t^*} a(t-s)(z^3(t)-1)ds\right)$$

$$\geqslant -\frac{1}{\mu} \int_{-\infty}^{t^*} a(t-s)(z^3(t)-1)$$

If $z(t) - 1$ is sufficiently small, this gives an inequality having the form

$$\left(\frac{d}{dt}\right)_+ z(t) \geq - Ce^{-kt}(z-1)$$

It follows immediately that $\lim_{t \to +\infty} z(t) > 1$.

We conclude this chapter with an argument showing that theorem 3.1 does not hold, if the condition that $f(t) = 0$ for $t > t_0$ is replaced by exponential decrease of f and $\alpha \neq 1$ (in case $\alpha = 1$ the previous argument still goes through, the only difference being that $f(t) \sum_{i=1}^{N} (\alpha_i - \beta_i)$ has to be added on the right side of (3.1)). We restrict ourselves to the case $N = 1$. Now (2.1) reads:

$$-\mu\dot{y} = gy^3 - h - f(t)y^\alpha$$

$$\dot{g} = -\lambda g + \frac{K}{y^2}$$

$$\dot{h} = -\lambda h + Ky$$

We now solve these equations for $t \geq 0$ by the following ansatz:

$$y = y_0 e^{\nu t},\ g = g_0 e^{-2\nu t} + g_1 e^{-\lambda t},\ h = h_0 e^{\nu t} + h_1 e^{-\lambda t},$$

$$f = f_0 e^{(1-\alpha)\nu t} + g_1 y_0^{3-\alpha} e^{((3-\alpha)\nu-\lambda)t} - h_1 y_0^{-\alpha} e^{(-\alpha\nu-\lambda)t}.$$

After some calculation one finds that this satisfies the equations if

$$g_0 = \frac{K}{y_0^2(\lambda-2\nu)},\ h_0 = \frac{Ky_0}{\lambda + \nu} \quad \text{and}$$

$$f_0 y_0^{\alpha-1} = \frac{3\nu K + \mu\nu(\lambda-2\nu)(\lambda+\nu)}{(\lambda-2\nu)(\lambda+\nu)}$$

We thus find solutions where f goes to zero exponentially, but $y \to \infty$ for $\alpha > 1$ and $y \to 0$ for $\alpha < 1$.

All we have to make sure is that by appropriate continuation for $t < 0$ we can match the conditions at $t = -\infty$. For this purpose continue y in an arbitrary way to the left such that y is smooth and approaches 1 exponentially at $t = -\infty$. The equations for g and h respectively then have unique solutions approaching $\frac{K}{\lambda}$ for $t \to -\infty$. These solutions can be matched to the solutions for $t > 0$ by appropriate choice of g_1 and h_1. Finally f is determined by the first equation.

4. THE CASE $\mu = 0$

In this case the first equation of (2.1) becomes

$$y^3 \cdot \sum_{i=1}^{N} g_i - \sum_{i=1}^{N} h_i - f(t)y^\alpha = 0.$$

PROPOSITION 4.1 For any $g > 0$, $h > 0$ and $0 < \alpha < 3$ the equation $F(y) = gy^3 - h - fy^\alpha = 0$ has a unique solution in $(0,\infty)$.

PROOF: We have $F(0) < 0$, $\lim_{y\to\infty} F(y) > 0$, so there is clearly a positive solution. To show it is unique, we investigate zeros of $F'(y)$. We have $F'(y) = 3gy^2 - \alpha fy^{\alpha-1}$. If $y > 0$ and $F'(y) = 0$, we find $F(y) = \frac{1}{\alpha}yF'(y) + y^3(1 - \frac{3}{\alpha})g - h < 0$. This means F cannot have a positive maximum, whence the result.

The solution $y(g,h,f)$ can then be inserted into the other equations, yielding a system of $2N$ equations.

THEOREM 4.1 The same statement as in theorem 3.1 holds also for $\mu = 0$. Also, Corollary 3.2 still holds.

SKETCH OF THE PROOF: The existence of a solution on $(-\infty,-t_1)$ and global existence in time are proved in the same manner as before, and we do not repeat the arguments. If $f = 0$, one now finds from (2.2)

$$\dot{\gamma}_i = -\lambda_i\gamma_i + K_i + 2\gamma_i\frac{\dot{Y}}{y}$$

$$\dot{\delta}_i = -\lambda_i\delta_i + K_i - \delta_i\frac{\dot{Y}}{y}$$

This leads to

$$\sum_{i=1}^{N}\left(\frac{1}{2}\ \frac{\alpha_i\dot{\alpha}_i}{\alpha_i + \frac{K_i}{\lambda_i}} + \frac{\dot{\beta}_i\beta_i}{\beta_i + \frac{K_i}{\lambda_i}}\right) = -\sum_{i=1}^{N}\left(\frac{\lambda_i\alpha_i^{\ 2}}{2(\alpha_i + \frac{K_i}{\lambda_i})} + \frac{\lambda_i\beta_i^{\ 2}}{\beta_i + \frac{K_i}{\lambda_i}}\right) + \frac{\dot{Y}}{y}\cdot\sum_{i=1}^{N}(\alpha_i - \beta_i)$$

where α_i and β_i are defined as before.

Since $\sum_i(\alpha_i - \beta_i)$ is now equal to zero, we still find that α_i and β_i approach 0 exponentially, whence the result.

For the corollary, observe that

$$\dot{y}(t) \cdot 3y^2(t) \int_{-\infty}^{t} a(t-s) \frac{1}{y^2(s)} = - \int_{-\infty}^{t} a'(t-s) \left(\frac{y^3(t)}{y^2(s)} - y(s)\right) ds$$

Using this, one can apply an argument analogous to the previous one.

ACKNOWLEDGMENTS.

I thank J.A. Nohel and A.S. Lodge for helpful discussions, and I thank Deutsche Forschungsgemeinschaft for financial support.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) (14) MRC-TSR-2150

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER TSR# 2150	2. GOVT ACCESSION NO. AD-A096 667	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) Evolution of the Shape of a Polymer Subjected to a Force		5. TYPE OF REPORT & PERIOD COVERED Summary Report - no specific reporting period
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) M. Renardy (12)		8. CONTRACT OR GRANT NUMBER(s) (15) DAAG29-80-C-0041
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of Wisconsin 610 Walnut Street Madison, Wisconsin 53706		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit 1 (Applied Analysis)
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Research Office P.O. Box 12211 Research Triangle Park, North Carolina 27709		12. REPORT DATE (11) ~~December 1980~~
		13. NUMBER OF PAGES 13
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (9) Technical summary rept.		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

We consider the nonlinear Volterra integrodifferential equation

$$-\mu\dot{y} = \int_{-\infty}^{t} a(t-s)\left(\frac{y^3(t)}{y^2(s)} - y(s)\right)ds - f(t)y^\alpha$$

where $a(u) = \sum_{i=1}^{N} K_i e^{-\lambda_i u}$, $0 < \alpha < 3$, and $f\colon \mathbb{R} \to \mathbb{R}$ is a given function.

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20. ABSTRACT, continued

We study the existence of solutions and their asymptotic behavior as $t \to \infty$ for certain classes of functions f. The main result is that for each f satisfying $\lim_{t \to -\infty} e^{-\sigma t} f(t) = 0$ for some $\sigma > 0$ and $f(t) = 0$ for $t \geq t_0$ there exists a unique solution $y(t)$ satisfying $\lim_{t \to -\infty} y(t) = 1$. Moreover, for this solution $\lim_{t \to +\infty} y(t)$ exists. This holds for $\mu > 0$ and $\mu = 0$. It is further shown that the condition $f(t) = 0$ for $t \geq t_0$ can not be replaced by exponential decrease unless f is small or $\alpha = 1$.

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